Keynes's Treatise on Probability

Given as an Introductory lecture to a Turing forum for the centenary of Keynes's Treatise in 2021.

I am quite nervous about my credentials for introducing this centenary celebration, being neither a mathematician nor an economist. Keynes was both, have gone through the mathematics Tripos in King's College, ending as 12th Wrangler in 1905. However, according to Robert Skidelsky's authoritative biography, his heart was never really in mathematics, and he was much more engaged with philosophy, a preoccupation of most of his friends as well and a clear influence on the theory of his *Treatise*. So this brings him somewhat into my own neck of the woods.

Coming from a nonconformist background, and with an economist and mathematician for a father, he was naturally interested in social questions, and studied with the economist Alfred Marshall for a short time after his Tripos. He then joined the India Office, after passing the Civil Service examinations in 1906. The leisurely life of the Edwardian Civil Service gave him time to set about writing a dissertation on the logic of probability, aiming at a Fellowship at King's College. This dissertation was the nucleus of *The Treatise*. It gained him his Fellowship in 1909, after which the bulk of his time had to be spent teaching economics. The treatise itself was therefore not completed until 1914. Due to the intervention of the First World War it was only published in 1921, after Keynes made extensive changes to the first proofs. By this time Keynes was famous as a government advisor, a critic of the Treatise of Versailles, a collector, man of business, and along with Lytton Strachey a prominent member of Bloomsbury. Thereafter probability was only a sideline, although there is debate about the extent to which his scepticism about the subject permeates the later *General Theory.*

Apart from anything else, the *Treatise* is a phenomenal piece of scholarship, giving weighty discussions of virtually every European who had ever contributed to the theory of probability. Indeed Keynes concludes it with a bibliography of twenty five closely printed pages, which he introduces by saying that:

> The subject has preserved its mystery, and has thus attracted the notice, profound or, more often, casual, of the most speculative minds. Leibniz, Pascal Arnaud, Huygens, Sprinoza, Jacques and Daniel Bernoullie, Hume, D'Alembert, Condorcet, Euler, Laplace, Poisson, Cournot, Quetelet, Gauss, Mill, Boole, Tchebychev, Lexis, and Poincaré, to name those only who are dead, are catalogued below.

Keynes's positive view is that the theory of probability gets its subject matter by locating a rational relationship of partial implication, holding between some evidence, *p,* and some conclusion, *q.* This relationship is logically analogous to full implication, which at the time was of course being brilliantly illuminated by the advances made in formal logic by Frege and Russell. Keynes shared their objective and indeed Platonic views of mathematics and logic as themselves describing abstract objects.

Since the *Treatise* is sufficiently replete with algebra to make much of it tough going, one might jump to the conclusion that it is essentially a mathematical work. But this is not so. Far more often the algebra is given in order to support scepticism about what can be achieved via mathematical methods. Keynes perhaps imbibed the limitations of purely numerical reasonings from his economic tutor. Alfred Marshall famously said

> "..I went more and more on the rules —(1) Use mathematics as a shorthand language rather than as an engine of enquiry (2) Keep to them until you have done (3) Translate into English (4) Illustrate by examples which are important in real life (5) Burn the mathematics (6) If you cannot succeed in (4), burn (3).

Keynes constantly pillories those who, like Laplace or Quetelet, supposed that a priori, mathematical reasoning by itself, could conjure up probabilities. His scepticism is laid out early on, in the third chapter where he insists that

> We can say that one thing is more like a second object than it is like a third; but there will very seldom be any meaning in saying that it is twice as like. Probability is, as far as measurement is concerned, closely analogous to similarity p. 28.

The book is peppered with similar statements:

> The old assumption that all quantity is numerical and that all quantitative characteristics are additive can no longer be sustained. Mathematical reasoning now appears as an aid in its symbolic rather than its numerical character. I, at any rate, have not the same lively hope as Condorcet or even as Edgeworth of "illuminating the moral and political sciences by the torch of algebra" (p. 316)

At the beginning of Part III, on Induction and Analogy he quotes Hume:

"nothing so like as eggs, yet no one, on account of this apparent similarity expects the same taste and relish in all of them." p. 217.

> It would be very surprising, in fact, if logic could tell us exactly how many instances we want , to yield us a given degree o certainty in empirical arguments…Cooly considered, this is a preposterous claim, which would have been universally rejected long ago, if those who made it had not so successfully concealed themselves from the eyes of common sense in a maze of mathematics. (pp. 388—9)

Perhaps my own favourite example of his scepticism comes when he talks about the consilience of evidence, as when two independent pieces of testimony each offer the same fact:

> The theory of "Testimony" itself, the theory that is to say of the combination of the evidence of witnesses, has occupied so considerable a space in the traditional treatment of probability that it will be worth while to examine it briefly. It may, however, be safely said that the principal conclusions on the subject, set out by Condorcet, Laplace, Poisson, Cournot and Boole, are demonstrably false. (p. 180)

In the final section of the book he pours equally cold water on the use of statistical methods:

> I do not myself believe that there is any direct and simple way method by which we can make the transition from an observed numerical frequency to a numerical measure of probability...In the next chapters we will consider the application of general inductive methods to this problem, and in this we will endeavour to discredit the mathematical charlatanry by which, for a hundred years past, the basis of theoretical statistics has been greatly undermined" p. 367.

His particular *bête noire* is the deployment of antecedent or a priori principles such as the principle of insufficient reason in advance of considering statistics, saying of one argument of Karl Pearson:

> "If we seek to discover what proportion of the population suffer from a certain disease or have red hair or are called Jones, it is preposterous to suppose that the proportion is as likely *a priori* to exceed as to fall short of (say) 50%...it invests any positive conclusion, which it is employed to support with far too high a degree of probability. Indeed this is so foolish a theorem that to entertain it is discreditable" pp. 381–2.

The last chapter is pessimistic about using "collections of facts for the prediction of future frequencies and associations."

By the end of the *Treatise* it looks to modern eyes as though one last push might have brought Keynes to the point of scepticism about his logical quasi-numerical probability relations altogether. This last push was provided by Ramsey, whose flat denial of the existence of these relations was first voiced (in a review in February 1922 edition of Ogden's *Cambridge* Magazine) directly after Keynes's book came out, and then repeated at length in his 1926 paper 'Truth and Probability'. Curiously enough, by avoiding these relations and replacing them by personal credence or degrees of belief in propositions, Ramsey actually put the epistemology of probability on a more secure footing.

Part of the reason for this is that any relation between evidence and conclusion of a non-deductive argument is only remotely analogous to logical entailment. Firstly it is not monotonic, so whereas if P entails Q then P & R entails Q, this is not so for support. New evidence may or may not augment or diminish any support old evidence gave to an hypothesis. Secondly since this is so such relations alone affords no ground for detachment, deriving what we really want, namely a degree of confidence in Q, from the relation itself. Finally whether one belief, or set of beliefs supports some conclusion is not a formal matter, whereas logical implication is. That is, whereas logical form is the same for any substitutions of non-logical predicates or names, support relations are not.

By what we might call humanizing it all, Ramsey opens up more epistemological possibilities. So for example, one of Keynes's important targets is the inversion of Bernoulli's theorem, whereby instead of deriving confidence in some frequency of results given a sequence in which each event has the same independent probability, we try to derive the probability from the observed frequency. As part of his campaign against the mathematicians (especially Karl Pearson) Keynes rightly says that there is no mathematical or logical relation between the observation and the probability (frequencies do not tell us that we have a Bernoulli sequence at all). But often enough there is clearly a good inductive reason for taking such a frequency as a guide to the future, which will be all Ramsey requires for a degree of confidence, increasing as the sequence grows longer and stabilizes around a frequency.

The shift from Russell's logicism to Ramsey's expressivism has been disguised by the unfortunate title of "subjectivism" or "the subjective theory of probability", first made common by Leonard Savage. It cannot be overemphasized that Ramsey is not in any sense a subjectivist. He knows we subject our own distributions of confidence to doubt and hope for improvement. His method for thinking about this derives from C. S. Peirce, asking what is the ideal habit of distributing confidence, and answering in a frequentist vein that we can praise or blame a habit of belief formation accordingly as the degree of belief it produces is near or far from the actual proportion of cases in which the habit leads to the truth. Ramsey, like

Keynes himself salutes Hume on induction but unlike Keynes denies that induction gives us a form of probable inference (p. 93 in Mellor's collection).

It is greatly to Keynes's credit that while he understood the force of Ramsey's teenage criticism he never wavered in his admiration for Ramsey. Patriotically I have to think this was a pity, for it led him to snatch Ramsey from his natural spiritual home, Trinity, just as soon as he had graduated. Much more sadly, of course, King's had less than five years to enjoy the fruits of Keynes's piracy.